INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE
B.MATH - Second Year, Second Semester, 2023-24
Statistics - II, Midterm Examination, September 20, 2023
Answer all questions **Maximum Marks: 50** **Time: 3 hours**

1. Let $X_1, X_2, \ldots, X_n$ be i.i.d. $U(\mu - 0.5, \mu + 0.5)$, where $-\infty < \mu < \infty$. Let $X_{(1)} < X_{(2)} < \cdots < X_{(n)}$ be order statistics. Find
(a) $E(X_{(i)})$ and $Var(X_{(i)})$, $1 \leq i \leq n$; and
(b) $E(X_{(k)} - X_{(l)})$ and $Var(X_{(k)} - X_{(l)})$, $1 \leq l < k \leq n$.
(c) Define $\hat{\mu} = \sum_{i=k+1}^{n-k} X_{(i)}/(n-2k)$, where $1 \leq k < n/2$ is an integer. Find $E(\hat{\mu})$ and $Var(\hat{\mu})$. [4+4+7]

2. Suppose X_1, X_2, X_3, X_4 are i.i.d Poisson(λ), $\lambda > 0$ and let $Y = X_1 + X_2$, $Z = X_3 + X_4$.
(a) Find the conditional distribution of (X_1, X_2, X_3, X_4) given (Y, Z) and using it show that (Y, Z) is sufficient for λ.
(b) Show that (Y, Z) is not minimal sufficient for λ. [5+5]

3. Suppose $X_1, X_2, \ldots, X_m$ and $Y_1, Y_2, \ldots, Y_n$ are independent random samples, respectively, from $N(2\mu, 10^2)$ and $N(\mu, 5^2)$, where $-\infty < \mu < \infty$ is the unknown parameter of interest.
(a) Find minimal sufficient statistic for μ. Is it complete?
(b) Find the MLE and UMVUE of μ.
(c) Find the Fisher's Information number, $I(\mu)$, for the combined sample. [4+6+5]

4. For observations $Y_1, \ldots, Y_n$, consider the linear model

$$Y_i = \beta_0 + \beta_1 x_i + \epsilon_i, \quad i = 1, \ldots, n,$$

where x_i is the value of a co-variate corresponding to Y_i and ϵ_i are i.i.d. errors having the $N(0, \sigma^2)$ distribution. Here β_0, β_1 and $\sigma^2 > 0$ are unknown parameters and x_i are treated as known constants.
(a) Show that the distribution of $Y_1, \ldots, Y_n$ belongs to k-variate exponential family. Find k.
(b) Find minimal sufficient statistics for $(\beta_0, \beta_1, \sigma^2)$. Is it complete? [5+5]